The form of the three given equations shows that α, β, γ are the three roots of the equation

$$\frac{x}{a+\varsigma}+\frac{y}{b+\varsigma}+\frac{z}{c+\varsigma}=1,$$

in which ς is regarded as the unknown. On clearing of fractions, and arranging in the form of a cubic equation in ε, it is seen that the sum of the three roots is $-(a+b+c)+(x+y+z)$.

Hence $\alpha+\beta+\gamma=-(a+b+c)+(x+y+z)$, and $x+y+z=a+\alpha+b+\beta+c+\gamma$.

NOTE. It may be of interest to state that if each letter be squared the result expresses the distance of any point from the origin in terms of ellipsoidal curvilinear coördinates.

156. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

$(z+x)a-(z-x)b=2yz....(1)$; $(x+y)b-(x-y)c=2xz....(2)$; $(y+z)c-(y-z)a=2xy....(3)$. Find the values of x, y, and z by the method of linear simultaneous equations.

Solution by G. B. M. ZERR, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let $x=\frac{1}{2}(b+c)u$, $y=\frac{1}{2}(a+c)v$, $z=\frac{1}{2}(a+b)w$.

$\therefore (a-b)w+(b+c)u=(a+c)vw....(1)$.

$(b-c)u+(a+c)v=(a+b)uw....(2)$.

$(c-a)v+(a+b)w=(b+c)uv....(3)$.

We might eliminate v, w and get an equation of the fifth degree in u. We will, however, proceed as follows: Add (1), (2), (3), then

$$aw(2-u-v)+bu(2-v-w)+cv(2-u-w)=0.$$

This is the case when $u=v=w=0$; or $u=v=w=1$; or $u=0$, $w=v=2$; or $v=0$, $u=w=2$; $w=0$, $u=v=2$.

The first two sets of values satisfy the conditions.

$\therefore x=y=z=0$; $x=\frac{1}{2}(b+c)$, $y=\frac{1}{2}(a+c)$, $z=\frac{1}{2}(a+b)$.

NOTE. This is exercise 31, page 224, Systems of Linear Simultaneous Equations, of Fisher and Schwatt's *Higher Algebra*, and has given teachers of algebra throughout the country considerable trouble. Solving the equations for a, b, and c, we readily find that

$$a=-x+y+z,$$
$$b=x-y+z, \text{ and}$$
$$c=x+y-z.$$

$\therefore x=\frac{1}{2}(b+c)$, $y=\frac{1}{2}(a+c)$, $z=\frac{1}{2}(a+b)$, as one set of values for x, y, and z. EDITOR F.

Also solved by *L. C. WALKER.*